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Influence of CP and CPT on production and decay of Dirac and Majorana fermions

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Abstract. The consequences of CP and CPT invariance for production and subsequent decay of Dirac and Majorana fermions in polarized fermion–antifermion annihilation are analytically studied. We derive general symmetry relations for the production spin density matrix and for the three-particle decay matrices and obtain constraints for the polarization and the spin–spin correlations of Dirac and Majorana fermions. We prove that only for Majorana fermions the energy and opening angle distribution factorizes exactly into contributions from production and decay if CP is conserved.

1 Introduction

It is generally accepted that supersymmetry (SUSY) is one of the most promising concepts for physics beyond the standard model (SM). A special feature of SUSY models is the Majorana character of the neutralinos, the fermionic superpartners of the neutral gauge and Higgs bosons. After the observation of candidates for neutralinos at a future e^+e^- collider [1], their identification as Majorana particles will be indispensable. Therefore an extensive study of the general characteristics of Majorana fermions produced in e^+e^- annihilation is of particular interest.

In [2,3], the authors analyzed the consequences of CP and CPT invariance for the symmetry properties of the cross section for Majorana fermions produced in e^+e^- annihilation with polarized beams. They proposed useful methods to distinguish between Majorana and neutral Dirac particles via the energy distributions of the leptons from their leptonic decay.

In the present paper we extend the investigation of CP and CPT symmetry properties to the complete spin production density matrix for Dirac and Majorana fermions and to their decay matrices for a three-body decay. We consider the most general dependence on beam polarization so that our analysis is also applicable to $\mu^+\mu^-$ annihilation with the exchange of Higgs bosons.

CP and CPT symmetry relations lead to important consequences for the factorization of differential cross sections for production and subsequent decay into a production and a decay piece. In [4] it has been proven that the differential cross section factorizes only if the kinematic variables are properly chosen. However, factorization is ruled out in particular for the energy and angular distributions of the decay products in the lab frame due to interference between the helicity amplitudes.

In numerical analyses [5] it was demonstrated that for production and three-particle decay of neutralinos in e^+e^- annihilation the energy distribution of the decay products in the lab frame as well as the distribution of the opening angle between two of them indeed factorize. Contrary, for production and three-particle decay of charginos [6] the spin correlations between production and decay considerably contribute to the energy distribution of the decay products and therefore prevent factorization. Here we prove that the factorization of these observables in the case of Majorana fermions is a consequence of their CP/CPT symmetry properties. We show that the analysis of both the energy spectrum of the decay fermions and the opening angle distribution is useful for the identification of the Majorana character of neutral particles produced in fermion–antifermion annihilation with polarized beams.

This paper is organized as follows. In Sect. 2.1 we give details of the spin density matrix formalism which is applied to the analysis of the consequences of CPT and CP invariance for the production density matrix of two different Dirac and Majorana fermions in Sect. 2.2 and for three-body decay matrices in Sect. 2.3. In Sect. 3 we study analytically the consequences for angular distributions and the energy spectra of the decay fermions.

2 Constraints on the production of Dirac and Majorana fermions from CPT and CP

2.1 Spin density matrix and cross section

We consider pair production of Dirac or Majorana fermions in fermion–antifermion annihilation and their sub-

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sequent three-particle decay. The helicity amplitudes for the production processes

$$f(\vec{p}_1,\lambda_1)\bar{f}(\vec{p}_2,\lambda_2)\to f_i(\vec{p}_i,\lambda_i)\bar{f}_j(\vec{p}_j,\lambda_j) \qquad (1)$$

are denoted by $T^{\lambda_i\lambda_j}_{\mathrm{P},\lambda_1\lambda_2}$. In the case of Majorana particles it is $\bar{f}_j\equiv f_j$. For the decay processes

$$f_i(\vec{p}_i,\lambda_i)\to f_{i1}(\vec{p}_{i1})f_{i2}(\vec{p}_{i2})f_{i3}(\vec{p}_{i3}), \qquad (2)$$

$$f_j(\vec{p}_j,\lambda_j)\to f_{j1}(\vec{p}_{j1})f_{j2}(\vec{p}_{j2})f_{j3}(\vec{p}_{i3}) \qquad (3)$$

the helicity amplitudes are given by T_{D,λ_i} and T_{D,λ_j}. Here f_{i1} (f_{j1}) is a charged or neutral Dirac or a Majorana fermion and f_{i2} (f_{j2}), f_{i3} (f_{j3}) are Dirac fermions. The notation of the decays (2) and (3) does not distinguish between fermions and antifermions for the initial particles and their decay products. The helicities of the decay products are also suppressed.

The spin density matrices of the polarized beams can be written as

$$\rho(f)=\frac{1}{2}(1+P^i_f\sigma^i), \qquad (4)$$

$$\rho(\bar{f})=\frac{1}{2}(1+P^i_{\bar{f}}\sigma^i), \qquad (5)$$

where P^1_f, P^2_f, P^3_f ($P^1_{\bar{f}}$, $P^2_{\bar{f}}$, $P^3_{\bar{f}}$) is the transverse polarization of f ($\bar{f}$) in the production plane, the polarization which is perpendicular to the production plane and the longitudinal polarization.

The amplitude squared of the combined process of production and decay reads

$$|T|^2=\sum_{\lambda_i\lambda_j\lambda'_i\lambda'_j}|\Delta(f_i)|^2|\Delta(f_j)|^2\rho_{\mathrm{P}}^{\lambda_i\lambda_j,\lambda'_i\lambda'_j}\rho_{\mathrm{D},\lambda'_i\lambda_i}\rho_{\mathrm{D},\lambda'_j\lambda_j}. \qquad (6)$$

It is composed of the (unnormalized) spin density production matrix

$$\rho_{\mathrm{P}}^{\lambda_i\lambda_j,\lambda'_i\lambda'_j}=\sum_{\lambda_1\lambda_2\lambda'_1\lambda'_2}\rho(f)_{\lambda'_1\lambda_1}\rho(\bar{f})_{\lambda'_2\lambda_2}T^{\lambda_i\lambda_j}_{\mathrm{P},\lambda_1\lambda_2}T^{\lambda'_i\lambda'_j *}_{\mathrm{P},\lambda'_1\lambda'_2}, \qquad (7)$$

the decay matrices

$$\rho_{\mathrm{D},\lambda'_i\lambda_i}=T_{\mathrm{D},\lambda_i}T^*_{\mathrm{D},\lambda'_i}, \qquad (8)$$

$$\rho_{\mathrm{D},\lambda'_j\lambda_j}=T_{\mathrm{D},\lambda_j}T^*_{\mathrm{D},\lambda'_j}, \qquad (9)$$

and the propagators

$$\Delta(f_k)=1/[p_k^2-m_k^2+\mathrm{i}m_k\Gamma_k], \quad k=i,j. \qquad (10)$$

Here p_k^2, m_k and Γ_k denote the four-momentum squared, mass and total width of the fermion f_k. For these propagators we use the narrow-width approximation.

We introduce for f_i (f_j) three spacelike polarization vectors $s^{a\mu}(f_i)$ ($s^{b\mu}(f_j)$) which together with p_i^μ/m_i (p_j^μ/m_j) form an orthonormal set. Then the spin density matrix of production and the decay matrices can be expanded in terms of Pauli matrices σ^a with the first (second) row and column corresponding to the helicity $\lambda=1/2$ ($-1/2$) [7]:

$$\begin{aligned}\rho_{\mathrm{P}}^{\lambda_i\lambda_j,\lambda'_i\lambda'_j}&=\delta_{\lambda_i\lambda'_i}\delta_{\lambda_j\lambda'_j}P(f_if_j)+\delta_{\lambda_j\lambda'_j}\sum_{a=1}^3\sigma^a_{\lambda_i\lambda'_i}\Sigma^a_{\mathrm{P}}(f_i)\\&+\delta_{\lambda_i\lambda'_i}\sum_{b=1}^3\sigma^b_{\lambda_j\lambda'_j}\Sigma^b_{\mathrm{P}}(f_j)\\&+\sum_{a,b=1}^3\sigma^a_{\lambda_i\lambda'_i}\sigma^b_{\lambda_j\lambda'_j}\Sigma^{ab}_{\mathrm{P}}(f_if_j),\end{aligned} \qquad (11)$$

$$\rho_{\mathrm{D},\lambda'_i\lambda_i}=\delta_{\lambda'_i\lambda_i}D(f_i)+\sum_{a=1}^3\sigma^a_{\lambda'_i\lambda_i}\Sigma^a_{\mathrm{D}}(f_i), \qquad (12)$$

$$\rho_{\mathrm{D},\lambda'_j\lambda_j}=\delta_{\lambda'_j\lambda_j}D(f_j)+\sum_{b=1}^3\sigma^b_{\lambda'_j\lambda_j}\Sigma^b_{\mathrm{D}}(f_j). \qquad (13)$$

Here a (b) $=1,2,3$ refers to the polarization vectors of f_i (f_j). The contributions $\Sigma^a_{\mathrm{P}}(f_i)$ ($\Sigma^b_{\mathrm{P}}(f_j)$), $\Sigma^a_{\mathrm{D}}(f_i)$ ($\Sigma^b_{\mathrm{D}}(f_j)$) are linear, and $\Sigma^{ab}_{\mathrm{P}}(f_if_j)$ is bilinear in the polarization vectors $s^{a\mu}(f_i)$ ($s^{b\nu}(f_j)$). In (11) the dependence of ρ_{P} on beam polarization has been suppressed.

The polarization vectors $\vec{s^1}$, $\vec{s^2}$, $\vec{s^3}$ form an orthogonal right–handed system in the lab system:

(1) $\vec{s^3}(f_i)$ ($\vec{s^3}(f_j)$) is in the direction of momentum $\vec{p_i}$ ($\vec{p_j}$),
(2) $\vec{s^2}(f_i)=(\vec{p}_1\times\vec{p_i})/(|\vec{p}_1\times\vec{p_i}|)=\vec{s^2}(f_j)$ is perpendicular to the production plane,
(3) $\vec{s^1}(f_i)$ ($\vec{s^1}(f_j)$) is in the production plane orthogonal to the momentum $\vec{p}_i$ ($\vec{p}_j$).

Then $\Sigma^3_{\mathrm{P}}(f_{i,j})/P(f_if_j)$ is the longitudinal polarization, $\Sigma^1_{\mathrm{P}}(f_{i,j})/P(f_if_j)$ is the transverse polarization in the scattering plane and $\Sigma^2_{\mathrm{P}}(f_{i,j})/P(f_if_j)$ is the polarization perpendicular to the scattering plane. The terms $\Sigma^{ab}_{\mathrm{P}}(f_if_j)$ are due to correlations between the polarizations of both produced particles.

The amplitude squared $|T|^2$ of the combined process of production and decay (6) can be written as

$$\begin{aligned}|T|^2&=4|\Delta(f_i)|^2|\Delta(f_j)|^2\\&\times\Bigg[P(f_if_j)D(f_i)D(f_j)+\sum_{a=1}^3\Sigma^a_{\mathrm{P}}(f_i)\Sigma^a_{\mathrm{D}}(f_i)D(f_j)\\&+\sum_{b=1}^3\Sigma^b_{\mathrm{P}}(f_j)\Sigma^b_{\mathrm{D}}(f_j)D(f_i)\\&+\sum_{a,b=1}^3\Sigma^{ab}_{\mathrm{P}}(f_if_j)\Sigma^a_{\mathrm{D}}(f_i)\Sigma^b_{\mathrm{D}}(f_j)\Bigg],\end{aligned} \qquad (14)$$

and the differential cross section is

$$\mathrm{d}\sigma=\frac{1}{2s}|T|^2(2\pi)^4\delta^4\left(p_1+p_2-\sum_i p_i\right)\mathrm{dlips}, \qquad (15)$$

with the Lorentz invariant phase space element dlips.

For the case of neutralinos and charginos the complete analytical expressions for the production density matrix and for the decay matrices are given in different presentations in [5,8].

2.2 CPT and CP symmetries of the spin density matrix

In this section we derive constraints from CPT and CP invariance for the density matrix (11) for the production of two (charged or neutral) Dirac fermions and of two different Majorana fermions, respectively, for polarized beams.

All contributions from the exchange of particles α, β to the coefficients

$$\mathcal{E} = \{\mathrm{P}, \Sigma^a_{\mathrm{P}}, \Sigma^b_{\mathrm{P}}, \Sigma^{ab}_{\mathrm{P}}\} \tag{16}$$

of the spin density matrix (11) are composed of products $\mathcal{C}$ of couplings, the propagators $\Delta(\alpha)$, $\Delta(\beta)$ and complex functions $\mathcal{S}$ of momenta and polarization vectors:

$$\mathcal{E} \sim \mathrm{Re}\{\mathcal{C} \times \Delta(\alpha)\Delta(\beta)^* \times \mathcal{S}\}. \tag{17}$$

Assuming complex couplings and taking into account the finite widths of the exchanged particles the general structure is

$$\begin{aligned} \mathcal{E} \sim\ & \mathrm{Re}(\mathcal{C})\mathrm{Re}[\Delta(\alpha)\Delta(\beta)^*]\mathrm{Re}(\mathcal{S}) \\ & -\mathrm{Re}(\mathcal{C})\mathrm{Im}[\Delta(\alpha)\Delta(\beta)^*]\mathrm{Im}(\mathcal{S}) \\ & +\mathrm{Im}(\mathcal{C})\mathrm{Re}[\Delta(\alpha)\Delta(\beta)^*]\mathrm{Im}(\mathcal{S}) \\ & -\mathrm{Im}(\mathcal{C})\mathrm{Im}[\Delta(\alpha)\Delta(\beta)^*]\mathrm{Re}(\mathcal{S}), \qquad (18) \\ \equiv\ & \mathcal{E}_{RRR} + \mathcal{E}_{RII} + \mathcal{E}_{IRI} + \mathcal{E}_{IIR}. \qquad (19) \end{aligned}$$

If CP is conserved all couplings can be chosen real [9] and $\mathcal{E}_{IRI} = \mathcal{E}_{IIR} = 0$. The terms $\mathcal{E}_{RII}$ and $\mathcal{E}_{IIR}$ are due to interference between s-channel exchange of particles with finite width and the crossed channels. Their contributions can be neglected except in the vicinity of the s-channel resonances (exchange of gauge bosons in the case of e^+e^- or $q\bar{q}$ annihilation and in addition of Higgs bosons in $\mu^+\mu^-$ annihilation), since they are proportional to the width of the exchanged particles far from the pole. In $\mathcal{E}_{RII}$ and $\mathcal{E}_{IRI}$ the imaginary part of $\mathcal{S}$ originates from products of momenta and polarization vectors with the Levy–Civita tensor $\epsilon_{\mu\nu\rho\sigma}$. These two terms lead to triple product correlations of momenta which are sensitive to CP violation [3].

2.2.1 CPT invariance

Under CPT the helicity states of a Dirac fermion f_{D} and a Majorana fermion f_{M}, respectively, transform as

$$|f_{\mathrm{D}}(\vec{p},\lambda)\rangle \overset{CPT}{\to} (-1)^{\lambda-(1/2)}|\bar{f}_{\mathrm{D}}(\vec{p},-\lambda)\rangle, \tag{20}$$

$$|f_{\mathrm{M}}(\vec{p},\lambda)\rangle \overset{CPT}{\to} (-1)^{\lambda-(1/2)}\eta^{CPT}|f_{\mathrm{M}}(\vec{p},-\lambda)\rangle, \tag{21}$$

with the CPT phase $\eta^{CPT} = \pm\mathrm{i}$ of Majorana fermions [10]. Beyond that time reversal would interchange in- and outgoing states. However, if the finite width of the exchanged particles is neglected CPT invariance and the unitarity of the S-matrix leads in leading order perturbation theory to symmetry relations for the helicity amplitudes and for the production density matrix.

In the following we derive these symmetry relations from CPT invariance for (charged or neutral) Dirac fermions and for the special case of Majorana fermions.

2.2.1.1 Dirac fermions. CPT invariance relates in lowest order perturbation theory the spin density matrix $\rho_{\mathrm{P}}^{\lambda_i\lambda_j,\lambda'_i\lambda'_j}$ of the process

$$f\bar{f} \to f_i(\vec{p}_i,\lambda_i)\bar{f}_j(\vec{p}_j,\lambda_j) \tag{22}$$

to the spin density matrix $\bar{\rho}_{\mathrm{P}}^{\lambda_i\lambda_j,\lambda'_i\lambda'_j}$ of

$$f\bar{f} \to \bar{f}_i(\vec{p}_i,\lambda_i)f_j(\vec{p}_j,\lambda_j) \tag{23}$$

by

$$\begin{aligned} & \rho_{\mathrm{P}}^{\lambda_i\lambda_j,\lambda'_i\lambda'_j}(P^{1,2,3}_f; P^{1,2,3}_{\bar{f}}; \Theta) \\ & = \bar{\rho}_{\mathrm{P}}^{-\lambda_i-\lambda_j,-\lambda'_i-\lambda'_j}(P^{1,2}_{\bar{f}}, -P^3_{\bar{f}}; P^{1,2}_f, -P^3_f; \bar{\Theta})^*, \qquad (24) \end{aligned}$$

where Θ denotes the angle between f and f_i in (22) and $\bar{\Theta} \equiv \pi - \Theta$ is the angle between the direction of beam particle f and the outgoing antifermion $\bar{f}_i$ of the conjugated process (23) in the CMS. For polarized beams the polarizations of f and $\bar{f}$ are interchanged with sign reversal of the longitudinal polarization. To derive (24) the CPT transformation has been supplemented by a rotation $\mathcal{R}_2(\pi)$ around the normal to the production plane so that the beam direction is unchanged (see Fig. 1a).

Both density matrices ρ_{P} and $\bar{\rho}_{\mathrm{P}}$ can be expanded according to (11) with the coefficients $P(f_i\bar{f}_j)$, $\Sigma^a_{\mathrm{P}}(f_i)$, $\Sigma^b_{\mathrm{P}}(\bar{f}_j)$, $\Sigma^{ab}_{\mathrm{P}}(f_i\bar{f}_j)$ and $\bar{P}(\bar{f}_if_j)$, $\bar{\Sigma}^a_{\mathrm{P}}(\bar{f}_i)$, $\bar{\Sigma}^b_{\mathrm{P}}(f_j)$, $\bar{\Sigma}^{ab}_{\mathrm{P}}(\bar{f}_if_j)$. Then from CPT and $\mathcal{R}_2(\pi)$ invariance and with the substitutions

$$(P^{1,2,3}_f; P^{1,2,3}_{\bar{f}}; \Theta) \to (P^{1,2}_{\bar{f}}, -P^3_{\bar{f}}; P^{1,2}_f, -P^3_f; \bar{\Theta} = \pi - \Theta), \tag{25}$$

one obtains the following relations:

$$P(f_i\bar{f}_j) = \bar{P}(\bar{f}_if_j), \tag{26}$$

$$\Sigma^a_{\mathrm{P}}(f_i) = \eta_a\bar{\Sigma}^a_{\mathrm{P}}(\bar{f}_i), \tag{27}$$

$$\Sigma^b_{\mathrm{P}}(\bar{f}_j) = \eta_b\bar{\Sigma}^b_{\mathrm{P}}(f_j), \tag{28}$$

$$\Sigma^{ab}_{\mathrm{P}}(f_i\bar{f}_j) = \eta_a\eta_b\bar{\Sigma}^{ab}_{\mathrm{P}}(\bar{f}_if_j), \tag{29}$$

with $\eta_1 = 1 = \eta_2$ and $\eta_3 = -1$.

2.2.1.2 Majorana fermions. Since we have $\bar{\rho}_{\mathrm{P}} \equiv \rho_{\mathrm{P}}$ in (24) the invariance under $CPT \times \mathcal{R}_2(\pi)$ leads in lowest order perturbation theory to constraints for the production density matrix. The different terms in the expansion (11) of ρ_{P} can be classified into contributions which are symmetric (Type S) and antisymmetric (Type A) with regard to the substitution

$$(P^{1,2,3}_f; P^{1,2,3}_{\bar{f}}; \Theta) \to (P^{1,2}_{\bar{f}}, -P^3_{\bar{f}}; P^{1,2}_f, -P^3_f; \pi - \Theta), \tag{30}$$

where Θ denotes the angle between f and f_i in the CMS.

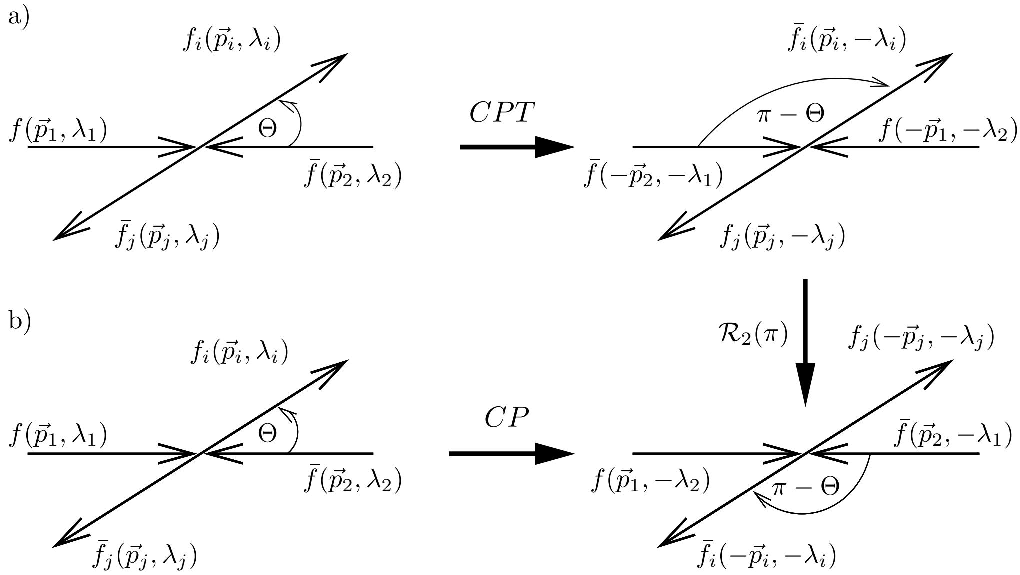

Fig. 1a,b. Production processes in the CMS under **a** CPT followed by a rotation $\mathcal{R}_2(\pi)$ and **b** CP transformation

(1) Type S: P, $\Sigma_{\rm P}^1$, $\Sigma_{\rm P}^2$, $\Sigma_{\rm P}^{11}$, $\Sigma_{\rm P}^{22}$, $\Sigma_{\rm P}^{33}$, $\Sigma_{\rm P}^{12}$, $\Sigma_{\rm P}^{21}$,
(2) Type A: $\Sigma_{\rm P}^3$, $\Sigma_{\rm P}^{13}$, $\Sigma_{\rm P}^{31}$, $\Sigma_{\rm P}^{23}$, $\Sigma_{\rm P}^{32}$.

All of them include terms $\mathcal{E}_{RRR}$ and $\mathcal{E}_{IRI}$ of (18).

For unpolarized beams the terms of type S are forward-backward symmetric, whereas those of type A are forward-backward antisymmetric. These symmetry properties hold also for production of Majorana fermions with polarized e^+e^- beams.

The forward–backward symmetry of the term P results in the FB symmetry of the differential cross sections for the production of Majorana fermions with unpolarized and longitudinally polarized beams [3]. Beyond that, also the symmetry properties of the different components of their polarization and of the spin–spin correlations are specific for their Majorana character.

2.2.2 CP invariance

We now study the consequences of the invariance under a CP transformation (illustrated in Fig. 1b) for the production of two Dirac or two Majorana fermions.

Under CP the helicity states of a Dirac and a Majorana fermion transform as

$$|f_{\rm D}(\vec{p}, \lambda)\rangle \stackrel{CP}{\to} |\bar{f}_{\rm D}(-\vec{p}, -\lambda)\rangle, \tag{31}$$

$$|f_{\rm M}(\vec{p}, \lambda)\rangle \stackrel{CP}{\to} \eta^{CP}|f_{\rm M}(-\vec{p}, -\lambda)\rangle, \tag{32}$$

with $\eta^{CP} = \pm {\rm i}$.

2.2.2.1 Dirac fermions. CP invariance results in a relation between the spin density matrix $\rho_{\rm P}$ for the conjugated process

$$f\bar{f} \to f_i(\vec{p}_i, \lambda_i)\bar{f}_j(\vec{p}_j, \lambda_j) \tag{33}$$

and the density matrix $\bar{\rho}_{\rm P}$ for

$$f\bar{f} \to \bar{f}_i(\vec{p}_i, \lambda_i)f_j(\vec{p}_j, \lambda_j), \tag{34}$$

$$\rho_{\rm P}^{\lambda_i\lambda_j,\lambda_i'\lambda_j'}(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta) = \bar{\rho}_{\rm P}^{-\lambda_i-\lambda_j,-\lambda_i'-\lambda_j'}(P_{\bar{f}}^1, -P_{\bar{f}}^{2,3}; P_f^1, -P_f^{2,3}; \bar{\Theta}), \tag{35}$$

with the same denotation as in (24). For polarized beams the polarizations of f and $\bar{f}$ are interchanged with an additional sign reversal for the longitudinal polarization and the polarization perpendicular to the production plane.

Expanding both density matrices $\rho_{\rm P}$ and $\bar{\rho}_{\rm P}$ according to (11) one obtains with the same substitutions

$$(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta) \to (P_{\bar{f}}^1, -P_{\bar{f}}^{2,3}; P_f^1, -P_f^{2,3}; \bar{\Theta} = \pi - \Theta) \tag{36}$$

as in (35), the following relations between the coefficients:

$$P(f_i\bar{f}_j) = \bar{P}(\bar{f}_if_j), \tag{37}$$

$$\Sigma_{\rm P}^a(f_i) = \eta_a\bar{\Sigma}_{\rm P}^a(\bar{f}_i), \tag{38}$$

$$\Sigma_{\rm P}^b(\bar{f}_j) = \eta_b\bar{\Sigma}_{\rm P}^b(f_j), \tag{39}$$

$$\Sigma_{\rm P}^{ab}(f_i\bar{f}_j) = \eta_a\eta_b\bar{\Sigma}_{\rm P}^{ab}(\bar{f}_if_j), \tag{40}$$

with $\eta_1 = +1$ and $\eta_2 = -1 = \eta_3$. For unpolarized or longitudinally polarized e^+e^- beams, the dependence on the beam polarization is given by the two factors $(1 - P_f^3P_{\bar{f}}^3)$ and $(P_f^3 - P_{\bar{f}}^3)$. If the widths of the exchanged particles in the s-channel are neglected, the CPT symmetry relations (24) hold and one obtains

$$\Sigma_{\rm P}^2 = 0, \quad \Sigma_{\rm P}^{12} = 0 = \Sigma_{\rm P}^{21}, \quad \Sigma_{\rm P}^{23} = 0 = \Sigma_{\rm P}^{32}. \tag{41}$$

2.2.2.2 Majorana fermions. With $\bar{\rho}_{\rm P} \equiv \rho_{\rm P}$ in (35) CP invariance again leads to a classification of the different contributions in the expansion (11) of $\rho_{\rm P}$ according to their CP symmetry properties with regard to the substitution

$$(P_f^{1,2,3}; P_{\bar{f}}^{1,2,3}; \Theta) \to (P_{\bar{f}}^1, -P_{\bar{f}}^{2,3}; P_f^1, -P_f^{2,3}; \pi - \Theta), \tag{42}$$

where Θ is the scattering angle in the CMS:

Table 1. The forward–backward symmetry (s) and antisymmetry (a) of all terms of the production spin density matrix (11) of Majorana fermions for unpolarized or longitudinally polarized e^+e^- beams with ($\Gamma_\alpha \neq 0$) and without ($\Gamma_\alpha = 0$) consideration of the width of the exchanged particles α for CP conservation (CP) or CP violation ($\not{C}\!\not{P}$)

CP, $\Gamma_\alpha = 0$
s: P, Σ_P^1, Σ_P^{11}, Σ_P^{22}, Σ_P^{33}
a: Σ_P^3, Σ_P^{13}, Σ_P^{31}
$\Sigma_\mathrm{P}^2 = 0$, $\Sigma_\mathrm{P}^{12} = 0 = \Sigma_\mathrm{P}^{21}$, $\Sigma_\mathrm{P}^{23} = 0 = \Sigma_\mathrm{P}^{32}$

	$\not{C}\!\not{P}$, $\Gamma_\alpha = 0$		CP, $\Gamma_\alpha \neq 0$
s:	P, Σ_P^1, Σ_P^2, Σ_P^{11}, Σ_P^{22}, Σ_P^{33}, Σ_P^{12}, Σ_P^{21}	s:	P, Σ_P^1, Σ_P^{11}, Σ_P^{22}, Σ_P^{33}, Σ_P^{23}, Σ_P^{32}
a:	Σ_P^3, Σ_P^{13}, Σ_P^{31}, Σ_P^{23}, Σ_P^{32}	a:	Σ_P^2, Σ_P^3, Σ_P^{12}, Σ_P^{21}, Σ_P^{13}, Σ_P^{31}

(1) Type S: P, Σ_P^1, Σ_P^{11}, Σ_P^{22}, Σ_P^{33}, Σ_P^{23}, Σ_P^{32},
(2) Type A: Σ_P^2, Σ_P^3, Σ_P^{12}, Σ_P^{21}, Σ_P^{13}, Σ_P^{31}.

All terms have contributions with the structure $\mathcal{E}_{RRR}$ and $\mathcal{E}_{RII}$ of (18).

For unpolarized and longitudinally polarized e^+e^- beams, the terms of type S are FB symmetric and those of type A are FB antisymmetric. For both Dirac and Majorana fermions the terms Σ_P^2, Σ_P^{12}, Σ_P^{21}, Σ_P^{23} and Σ_P^{32} vanish for unpolarized or longitudinally polarized beams if the CPT relations (26)–(29) hold. The FB symmetry or antisymmetry of the polarization and of the spin–spin correlations, however, is specific for the Majorana character of the produced fermions beyond the FB symmetry of their production cross section. Since in general the production of Dirac fermions will not exhibit these symmetry properties, the measurement of polarization and spin–spin correlations are helpful for an experimental verification of the Majorana character.

2.2.3 CP violation

If CP is violated due to complex couplings the two types $\mathcal{E}_{IRI}$, $\mathcal{E}_{IIR}$ of CP violating terms contribute in (18), whereas far from the s-channel resonances $\mathcal{E}_{IIR}$ can be neglected as explained in Sect. 2.2.

Since for Dirac fermions the CP transformation relates the two processes $f\bar{f} \to f_i\bar{f}_j$ and $f\bar{f} \to \bar{f}_i f_j$, we restrict ourselves to the discussion of the consequences of CP violation for the production density matrix of *Majorana fermions* for unpolarized or longitudinally polarized beams. Both types of CP violating terms, $\mathcal{E}_{IRI}$ and $\mathcal{E}_{IIR}$, lead to contributions which show different FB angular dependence compared to the CP conserving ones.

In particular, terms with the structure $\mathcal{E}_{IRI}$ result in nonvanishing polarization Σ_P^2 perpendicular to the production plane and nonvanishing spin–spin correlations $\Sigma_\mathrm{P}^{12,21}$ and $\Sigma_\mathrm{P}^{23,32}$:

(1) FB symmetric in $\mathcal{E}_{IRI}$: Σ_P^2, Σ_P^{12}, Σ_P^{21},
(2) FB antisymmetric in $\mathcal{E}_{IRI}$: Σ_P^{23}, Σ_P^{32}.

These terms are proportional to $\mathrm{Im}(\mathcal{S})$ and are sensitive to triple product correlations of momenta. They are CP-odd quantities [11].

All other contributions in (11) are CP-even quantities and get in the case of CP violation terms with the structure $\mathcal{E}_{IIR}$:

(1) FB symmetric in $\mathcal{E}_{IIR}$: Σ_P^3, Σ_P^{13}, Σ_P^{31},
(2) FB antisymmetric in $\mathcal{E}_{IIR}$: P, Σ_P^1, Σ_P^{11}, Σ_P^{22}, Σ_P^{33}.

The consequences of CP and CPT for the FB symmetry of the different terms of the production spin density matrix of Majorana fermions are summarized in Table 1.

2.3 CPT and CP symmetries of the decay matrices

In this section we derive from CPT and CP invariance constraints for the decay matrices for the three-body decay of neutral and charged Dirac fermions and of Majorana fermions.

2.3.1 CPT invariance

2.3.1.1 Dirac fermions. For the three-body decay

$$f_i(\vec{p}_i, \lambda_i) \to f_{i1}(\vec{p}_{i1}) f_{i2}(\vec{p}_{i2}) \bar{f}_{i3}(\vec{p}_{i3}) \tag{43}$$

of a Dirac fermion f_i into a Dirac fermion f_{i1} and a fermion–antifermion pair $f_{i2}\bar{f}_{i3}$, CPT invariance relates the decay matrix ρ_D of (43) with the decay matrix $\bar{\rho}_\mathrm{D}$ of the process

$$\bar{f}_i(\vec{p}_i, \lambda_i) \to \bar{f}_{i1}(\vec{p}_{i1}) \bar{f}_{i2}(\vec{p}_{i2}) f_{i3}(\vec{p}_{i3}). \tag{44}$$

Since we do not study the polarization of the decay fermions, we sum over their helicities and obtain

$$\rho_{\mathrm{D},\lambda_i'\lambda_i}(\vec{p}_{i2}, \vec{p}_{i3}) = (-1)^{\lambda_i - \lambda_i'} \bar{\rho}^*_{\mathrm{D},-\lambda_i'-\lambda_i}(\vec{p}_{i3}, \vec{p}_{i2}), \tag{45}$$

with the momenta of the fermion–antifermion pair interchanged and the signs of the helicities reversed.

Expanding ρ_D and $\bar{\rho}_\mathrm{D}$ according to (12) leads for the coefficients to

$$D(\vec{p}_{i2}, \vec{p}_{i3}) = \bar{D}(\vec{p}_{i3}, \vec{p}_{i2}), \tag{46}$$

$$\Sigma_\mathrm{D}^a(\vec{p}_{i2}, \vec{p}_{i3}) = -\bar{\Sigma}_\mathrm{D}^a(\vec{p}_{i3}, \vec{p}_{i2}), \quad a = 1, 2, 3. \tag{47}$$

2.3.1.2 Majorana fermions. For the decay

$$f_i(\vec{p}, \lambda_i) \to f_{i1}(\vec{p}_{i1}) f_{i2}(\vec{p}_{i2}) \bar{f}_{i2}(\vec{p}_{i3}) \quad (48)$$

of a Majorana fermion f_i into a Majorana fermion f_{i1} and a Dirac fermion–antifermion pair $f_{i2}\bar{f}_{i2}$ with $\bar{\rho}_{\mathrm{D}} \equiv \rho_{\mathrm{D}}$ in (45), CPT invariance leads to constraints for the decay matrix from which one obtains definite symmetry properties for the expansion coefficients of ρ_{D} in (12):

$$D(\vec{p}_{i2}, \vec{p}_{i3}) = D(\vec{p}_{i3}, \vec{p}_{i2}) \quad (49)$$

$$\Sigma^a_{\mathrm{D}}(\vec{p}_{i2}, \vec{p}_{i3}) = -\Sigma^a_{\mathrm{D}}(\vec{p}_{i3}, \vec{p}_{i2}), \quad a = 1, 2, 3, \quad (50)$$

if the momenta of the fermion–antifermion pair $f_{i2}\bar{f}_{i2}$ are exchanged.

2.3.2 CP invariance

Applying the space-inversion P to the decay (43) also the momentum of the decaying particle f_i is reversed. We therefore add a rotation $\mathcal{R}_{\vec{n}}(\pi)$ around the normal to the plane $(\vec{p}_i, \vec{p}_{i1})$. Then $\mathcal{P} \times \mathcal{R}_{\vec{n}}(\pi)$ is equivalent to a reflection at the plane $(\vec{p}_i, \vec{p}_{i1})$.

2.3.2.1 Dirac fermions. The combined transformation $CP \times \mathcal{R}_{\vec{n}}(\pi)$ leads for the decay of a Dirac fermion to the process

$$\bar{f}_i(\vec{p}_i, -\lambda_i) \to \bar{f}_{i1}(\vec{p}_{i1}) \bar{f}_{i2}(\hat{p}_{i2}) f_{i3}(\hat{p}_{i3}), \quad (51)$$

where $\hat{p}_{i2}$ and $\hat{p}_{i3}$ denotes the momentum $\vec{p}_{i2}$ and $\vec{p}_{i3}$, respectively, reflected at the plane $(\vec{p}_i, \vec{p}_{i1})$. The invariance under $CP \times \mathcal{R}_{\vec{n}}(\pi)$ relates the decay matrix ρ_{D} for (43) with the decay matrix $\bar{\rho}_{\mathrm{D}}$ for the antiparticle in (51):

$$\rho_{\mathrm{D},\lambda'_i\lambda_i}(\vec{p}_{i2}, \vec{p}_{i3}) = (-1)^{\lambda_i - \lambda'_i} \bar{\rho}_{\mathrm{D},-\lambda'_i -\lambda_i}(\hat{p}_{i3}, \hat{p}_{i2}). \quad (52)$$

In contrast to the CPT relation (45) the momenta of the fermion–antifermion pair are interchanged and additionally reflected at the plane $(\vec{p}_i, \vec{p}_{i1})$. For the coefficients in the expansion (12) one obtains

$$D(\vec{p}_{i2}, \vec{p}_{i3}) = \bar{D}(\hat{p}_{i3}, \hat{p}_{i2}), \quad (53)$$

$$\Sigma^2_{\mathrm{D}}(\vec{p}_{i2}, \vec{p}_{i3}) = \bar{\Sigma}^2_{\mathrm{D}}(\hat{p}_{i3}, \hat{p}_{i2}), \quad (54)$$

$$\Sigma^{1,3}_{\mathrm{D}}(\vec{p}_{i2}, \vec{p}_{i3}) = -\bar{\Sigma}^{1,3}_{\mathrm{D}}(\hat{p}_{i3}, \hat{p}_{i2}). \quad (55)$$

Combining these CP relations with the CPT properties (46) and (47) of the expansion coefficients leads to

(1) symmetric D, $\Sigma^{1,3}_{\mathrm{D}}$,
(2) antisymmetric Σ^2_{D},

under the reflection of the momenta $\vec{p}_{i2}$ and $\vec{p}_{i3}$ at the plane $(\vec{p}_i, \vec{p}_{i1})$.

2.3.2.2 Majorana fermions. For the decay (48) of a Majorana fermion with $\bar{\rho}_{\mathrm{D}} \equiv \rho_{\mathrm{D}}$ in (52), the combined transformation $CP \times \mathcal{R}_{\vec{n}}(\pi)$ constrains the decay matrix and their expansion coefficients when the momenta of the Dirac fermion f_{i2} and antifermion $\bar{f}_{i2}$ are interchanged and reflected at the plane of the Majorana fermions f_i, f_{i1}. Neglecting the width of the exchanged particles, CP and CPT invariance leads to the same symmetry properties of D, $\Sigma^{1,2,3}_{\mathrm{D}}$ as in the case of Dirac fermions under the reflection of the momenta $\vec{p}_{i2}$, $\vec{p}_{i3}$ at the plane $(\vec{p}_i, \vec{p}_{i1})$.

3 Energy and angular distributions of the decay products

In this section we study the influence of the polarization and of the spin–spin correlations on the energy spectrum and angular distributions in the lab frame at the production of Dirac and Majorana fermions and their subsequent three-particle decay into fermions. If the decay of only one of the produced fermions, e.g. f_i, is considered, we have in (14) $\Sigma^b_{\mathrm{D}}(f_j) \equiv 0$ and $D(f_j) \equiv 1$. Then the total cross section for the combined process of production and decay is given by $\sigma_{\mathrm{P}}(f\bar{f} \to f_i f_j) \times \mathrm{BR}(f_i \to f_{i1} f_{i2} f_{i3})$.

3.1 Energy spectrum and opening angle distribution of the decay leptons

We expect that in general the energy distribution of the decay particles in the lab system depends on the polarization of the decaying particle. In its rest frame the decay angular distribution is determined by the polarization with respect to a suitably chosen quantization axis. Since boosting in this direction into the lab frame the energy of a decay particle depends on the orientation of its momentum with respect to this axis, it also depends on the polarization of the decaying particle. The same argument applies to the distribution of the opening angle in the lab frame between the decay products.

In the following we show that due to the specific CPT/CP properties of Majorana fermions the energy distribution and the opening angle distribution of the decay fermions are independent of the polarizations of the decaying particle. In this case both distributions factorize into a production and a decay piece.

For our analysis we parametrize the phase space in the lab frame in such a way that it factorizes into the production and decay phase space.

3.1.1 Energy spectrum

For the energy distribution $\mathrm{d}\sigma/\mathrm{d}E_{i2}$ the phase space can be parametrized by the polar angle $\Theta_{i,i2}$ ($\Theta_{i,i3}$) between the momenta of f_i and f_{i2} (f_{i3}) and the azimuth $\Phi_{i,i2}$ ($\Phi_{i,i3}$) of f_{i2} (f_{i3}). All polar (azimuthal) angles are denoted by $\Theta_{\alpha\beta}$ ($\Phi_{\alpha\beta}$), where the first index denotes the polar axis:

$$\mathrm{d}\sigma = \mathcal{F}|T|^2 \sin\Theta \mathrm{d}\Theta \mathrm{d}\Phi \sin\Theta_{i,i2} \mathrm{d}\Theta_{i,i2} \mathrm{d}\Phi_{i,i2} \times \sin\Theta_{i,i3} \mathrm{d}\Theta_{i,i3} \mathrm{d}\Phi_{i,i3} \mathrm{d}E_{i2}, \quad (56)$$

where Θ is the production angle. For light fermions f_{i2} and f_{i3} $\mathcal{F}$ is given by

$$\mathcal{F} = \frac{q}{2^{11}(2\pi)^7 m_i \Gamma_i E_b^3} \times \frac{E_{i2}[m_i^2 - m_{i1}^2 - 2E_{i2}(E_i - q\cos\Theta_{i,i2})]}{[E_i - q\cos\Theta_{i,i3} - E_{i2}(1 - \cos\Theta_{i2,i3})]^2}. \quad (57)$$

E_b denotes the beam energy, E_α denotes the energy of the fermion f_α in the lab frame and $q = |\vec{p}_i| = |\vec{p}_j|$. The

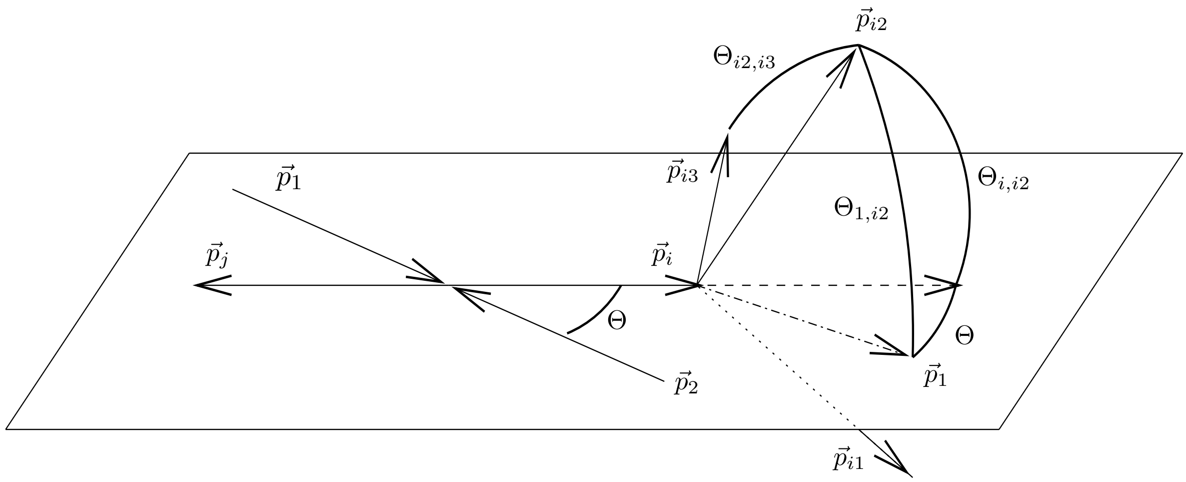

Fig. 2. Definition of momenta and polar angles in the lab system. The indices of the angles denote the plane covered by the corresponding momenta; the first index denotes the corresponding polar axis. The momenta and angles in the decay of f_j are denoted analogously

opening angle $\Theta_{i2,i3}$ between f_{i2} and f_{i3} can be expressed via (A.1) by the independent variables chosen in (56).

The kinematical boundaries for $\Theta_{i,i2}$ depend on E_{i2}:

$$\Theta_{i,i2} = \begin{cases} [0,\pi] \quad \text{for} \quad 0 < E_{i2} < \dfrac{m_i^2 - m_{i1}^2}{2(E_i+q)}, \\ \left[0, \arccos\left(\dfrac{E_i}{q} - \dfrac{m_i^2 - m_{i1}^2}{2qE_{i2}}\right)\right] \quad \text{for} \\ \qquad \dfrac{m_i^2 - m_{i1}^2}{2(E_i+q)} \le E_{i2} \le \dfrac{m_i^2 - m_{i1}^2}{2(E_i-q)}. \end{cases} \tag{58}$$

3.1.2 Opening angle distribution

For the distribution of the opening angle $\Theta_{i2,i3}$ between the fermions f_{i2} and f_{i3} from the decay $f_i \to f_{i1} f_{i2} f_{i3}$ it is favorable to parametrize the phase space of f_{i3} by $\Theta_{i2,i3}$ and the corresponding azimuthal angle $\Phi_{i2,i3}$ (Fig. 2), i.e. to substitute in (56) $\Theta_{i,i3} \to \Theta_{i2,i3}$ and $\Phi_{i,i3} \to \Phi_{i2,i3}$. The kinematical region of E_{i2} depends on $\Theta_{i,i2}$: $0 \le E_{i2} \le (m_i^2 - m_{i1}^2)/(2(E_i - q\cos\Theta_{i,i2}))$. The angles $\Theta_{i,i3}$ and $\Phi_{i,i3}$ can be expressed by (A.3), (A.6), and (A.8)–(A.10) by the chosen independent angular variables.

3.1.3 Transverse polarizations

In the contributions $\Sigma_{\mathrm{P}}^a(f_i)\Sigma_{\mathrm{D}}^a(f_i)$, $a = 1,2,3$, from the polarization of the decaying particle f_i both factors depend on the polarization vectors $s^{a\mu}(f_i)$ defined with respect to the production plane, cf. Section 2.1. They can be written

$$\Sigma_{\mathrm{P}}^a(f_i) = \tilde{\Sigma}_{\mathrm{P}}^\mu(f_i) s_\mu^a(f_i) \quad \text{and} \quad \Sigma_{\mathrm{D}}^a(f_i) = \tilde{\Sigma}_{\mathrm{D}}^\mu s_\mu^a(f_i). \tag{59}$$

To study their influence on the energy and the opening angle distribution we distinguish between the contributions $\Sigma_{\mathrm{P}}^{1,2}(f_i)$ from the transverse polarizations and $\Sigma_{\mathrm{P}}^3(f_i)$ from the longitudinal polarization. In the contributions of the transverse polarizations, the coefficients $\Sigma_{\mathrm{D}}^{1,2}(f_i)$ depend on the azimuth $\Phi_{i,i2}$ between the production plane and the plane defined by the decaying fermion f_i and the decay product f_{i2}. To separate the $\Phi_{i,i2}$ dependence we introduce a new system of polarization vectors $t^{a\nu}(f_i)$ with $t^{3\nu}(f_i) = s^{3\nu}(f_i)$, whereas in the lab system $\vec{t^2}(f_i)$ is perpendicular to the plane defined by the momenta of f_i and f_{i2} and $\vec{t^1}(f_1)$ is in this plane orthogonal to the momentum of the decaying fermion f_i. Then in (59) the transverse polarization vectors $s^{1,2\nu}(f_i)$ are

$$s^{1\nu}(f_i) = \cos\Phi_{i,i2} t^{1\nu}(f_i) - \sin\Phi_{i,i2} t^{2\nu}(f_i), \tag{60}$$

$$s^{2\nu}(f_i) = \sin\Phi_{i,i2} t^{1\nu}(f_i) + \cos\Phi_{i,i2} t^{2\nu}(f_i). \tag{61}$$

Applying the transformations (A.1) and (A.3), (A.6) and (A.8)–(A.10) for the angles $\Theta_{i,i3}$ and $\Phi_{i,i3}$ to the phase space parametrization (56) the contributions of $\Sigma_{\mathrm{D}}^{1,2}(f_i)$ to the opening angle distribution and to the lepton energy spectrum vanish for both Majorana and Dirac fermions due to the integration over $\Phi_{i,i2}$ [12].

3.1.4 Longitudinal polarization

3.1.4.1 Majorana fermions. The longitudinal polarization $\Sigma_{\mathrm{P}}^3(f_i)$ is forward–backward antisymmetric if CP is conserved, cf. Section 2.2.2. Due to the factorization of the phase space in production and decay also the contribution of the longitudinal polarization vanishes after integration over the production angle Θ.

Consequently both the energy and the opening angle distribution of the decay products of Majorana fermions in the laboratory system are independent of spin correlations and factorize exactly into production and decay if CP is conserved.

The factorization of the energy and opening angle distribution is essentially equivalent since for both observables it is a consequence of the FB antisymmetry of the longitudinal polarization of Majorana fermions.

Assuming CP violation and taking into account the widths of the exchanged particles the exact factorization of the energy distribution and of the opening angle distribution of the decay products of Majorana fermions is destroyed, since the longitudinal polarization Σ_P^3 gets FB symmetric contributions from CP violating terms. These additional terms have the structure $\mathcal{E}_{IIR}$, and are therefore for energies far from the resonance proportional to the width of the exchanged particle.

3.1.4.2 Dirac fermions. In general, the longitudinal polarization of produced Dirac fermions is not forward–backward asymmetric. Therefore the energy spectra and opening angle distributions of the decay products do not factorize into production and decay.

3.2 Decay lepton angular distribution

For the distribution $\mathrm{d}\sigma/\mathrm{d}\cos\Theta_{1,i2}$ of the angle $\Theta_{1,i2}$ in the lab frame between the incoming fermion f and the fermion f_{i2} from the decay $f_i \to f_{i1} f_{i2} f_{i3}$ we parametrize the phase space by

$$\mathrm{d}\sigma = \mathcal{F}|T|^2 \sin\Theta \mathrm{d}\Theta \mathrm{d}\Phi \sin\Theta_{1,i2}\mathrm{d}\Theta_{1,i2}\mathrm{d}\Phi_{1,i2} \times \sin\Theta_{1,i3}\mathrm{d}\Theta_{1,i3}\mathrm{d}\Phi_{1,i3}\mathrm{d}E_{i2}, \quad (62)$$

and express all angles in $\mathcal{F}$ (57) and the azimuth $\Phi_{i,i2}$ in (60) and (61) by the independent variables in (62). The relations are given in (A.2), (A.4), (A.5) and (A.7) [12]. Thus the kinematic factor $\mathcal{F}$ depends explicitly on the scattering angle Θ and neither the contributions $\Sigma_D^{1,2}(f_i)$ of the transverse polarizations nor that of the longitudinal polarization $\Sigma_P^3(f_i)$ vanish due to phase space integration.

Consequently neither for Dirac fermions nor for Majorana fermions the decay lepton angular distribution in the lab frame factorizes in production and decay but depends sensitively on the spin correlations.

3.3 Siamese opening angle distribution

The siamese opening angle $\Theta_{j2,i2}$ denotes the angle between decay products f_{i2} and f_{j2} from the decay of different particles f_i and $\bar{f}_j$ (f_j in the case of Majorana fermions) in the lab frame. The distribution $\mathrm{d}\sigma/\mathrm{d}\cos\Theta_{j2,i2}$ is determined by the spin–spin correlations Σ_P^{ab} (14) between the decaying mother particles f_i and $\bar{f}_j$. Here it is favorable to parametrize both phase spaces for the decay of f_i and of $\bar{f}_j$ by the angles $\Theta_{i,i3}$, $\Phi_{i,i3}$, and $\Theta_{i,j2}$, $\Phi_{i,j2}$:

$$\mathrm{d}\sigma = \mathcal{F}\mathcal{G}|T|^2 \sin\Theta \mathrm{d}\Theta \mathrm{d}\Phi \sin\Theta_{i,j2}\mathrm{d}\Theta_{i,j2}\mathrm{d}\Phi_{i,j2} \times \sin\Theta_{i,j3}\mathrm{d}\Theta_{i,j3}\mathrm{d}\Phi_{i,j3}\mathrm{d}E_{j2} \times \sin\Theta_{i,i3}\mathrm{d}\Theta_{i,i3}\mathrm{d}\Phi_{i,i3} \sin\Theta_{j2,i2}\mathrm{d}\Theta_{j2,i2}\mathrm{d}\Phi_{j2,i2}\mathrm{d}E_{i2}, \quad (63)$$

where $\Theta_{i,j3}$, $\Phi_{i,j3}$ are defined with respect to the direction of f_i and $\Theta_{j2,i2}$, $\Phi_{j2,i2}$. $\mathcal{F}$ is given by (57) and

$$\mathcal{G} = \frac{1}{2^5(2\pi)^5 m_j \Gamma_j} \times \frac{E_{j2}[m_j^2 - m_{j1}^2 - 2E_{j2}(E_j - q\cos\Theta_{i,j2})]}{[E_j - q\cos\Theta_{i,j3} - E_{j2}(1-\cos\Theta_{j2,j3})]^2}. \quad (64)$$

Table 2. For the energy and the different angular distributions of the decay fermions in the lab frame the contributing polarizations and spin–spin correlations are specified. For CP conservation (CP) the polarization dependence is different for Majorana and Dirac fermions. If CP is violated ($\not{C}\not{P}$) all distributions depend on spin correlations

	Dirac fermions		Majorana fermions	
Decay distrib.	CP	$\not{C}\not{P}$	CP	$\not{C}\not{P}$
energy	Σ_P^3	Σ_P^3	–	Σ_P^3
opening angle	Σ_P^3	Σ_P^3	–	Σ_P^3
angular	$\Sigma_P^{1,2,3}$	$\Sigma_P^{1,2,3}$	$\Sigma_P^{1,2,3}$	$\Sigma_P^{1,2,3}$
siamese angle	$\Sigma_P^{11,22,33}$	Σ_P^{ab}	$\Sigma_P^{11,22,33}$	Σ_P^{ab}

Using the transformation formulae (A.1) and (A.11)–(A.16) all angles can be expressed by the independent variables chosen in (63).

With the same arguments as for the contributions from transverse polarization to the opening angle distribution one infers the following.

The siamese opening angle distribution factorizes neither for Majorana nor for Dirac fermions; it depends, however, only on the diagonal spin–spin correlations Σ_P^{11}, Σ_P^{22}, Σ_P^{33} if CP is conserved.

The polarizations of the produced fermions and the spin–spin correlations between these contributing to the energy and the different angular distributions of their decay products are listed in Table 2.

4 Conclusion

Assuming CPT and CP invariance we derived symmetry properties of the spin density matrix for production of Dirac and Majorana fermions in fermion–antifermion annihilation with polarized beams and for their three-particle decay matrices. Majorana fermions show specific forward–backward symmetry properties of their polarizations and the spin–spin correlations. In particular for the production of Majorana fermions with unpolarized or longitudinally polarized e^+e^- beams their longitudinal polarization in the CMS is forward–backward antisymmetric so that the energy distributions of the decay products and the distribution of the opening angle between them are exactly independent of spin correlations if CP is conserved and if the width of the exchanged particles can be neglected. Thus they factorize into a production and a decay piece, which allows one to study the dynamics of the decay of Majorana fermions independently of that of the production process. Since this factorization is specific for Majorana fermions it opens the possibility to establish experimentally the Majorana character by comparing the measured decay energy and opening angle distributions with Monte Carlo studies.

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Appendix

A Parametrization of phase space

For completeness we list all angular relations used for the different parametrizations of the phase space in Sect. 3.

$$\cos\Theta_{i2,i3} = \cos\Theta_{i,i2}\cos\Theta_{i,i3} + \sin\Theta_{i,i2}\sin\Theta_{i,i3}\cos(\Phi_{i,i2}-\Phi_{i,i3}), \tag{A.1}$$
$$= \cos\Theta_{1,i2}\cos\Theta_{1,i3} + \sin\Theta_{1,i2}\sin\Theta_{1,i3}\cos(\Phi_{1,i2}-\Phi_{1,i3}), \tag{A.2}$$
$$\cos\Theta_{i,i3} = \cos\Theta_{i,i2}\cos\Theta_{i2,i3} + \sin\Theta_{i,i2}\sin\Theta_{i2,i3}\cos\Phi_{i2,i3}, \tag{A.3}$$
$$= \cos\Theta\cos\Theta_{1,i3} + \sin\Theta\sin\Theta_{1,i3}\cos\Phi_{1,i3}, \tag{A.4}$$
$$\sin\Phi_{i,i3} = \frac{\sin\Theta_{1,i3}}{\sin\Theta_{i,i3}}\sin\Phi_{1,i3}, \tag{A.5}$$
$$= \sin\Phi_{i,i2}\cos(\Phi_{i,i2}-\Phi_{i,i3}) - \cos\Phi_{i,i2}\sin(\Phi_{i,i2}-\Phi_{i,i3}), \tag{A.6}$$
$$\cos\Phi_{i,i3} = \frac{-[\cos\Theta_{1,i3}\sin\Theta - \sin\Theta_{1,i3}\cos\Theta\cos\Phi_{1,i3}]}{\sin\Theta_{i,i3}}, \tag{A.7}$$
$$= \cos\Phi_{i,i2}\cos(\Phi_{i,i2}-\Phi_{i,i3}) + \sin\Phi_{i,i2}\sin(\Phi_{i,i2}-\Phi_{i,i3}). \tag{A.8}$$

In (A.6) and (A.8) one has to insert

$$\cos(\Phi_{i,i2}-\Phi_{i,i3}) = \frac{\cos\Theta_{i2,i3}\sin\Theta_{i,i2} - \cos\Theta_{i,i2}\sin\Theta_{i2,i3}\cos\Phi_{i2,i3}}{\sin\Theta_{i,i3}}, \tag{A.9}$$
$$\sin(\Phi_{i,i2}-\Phi_{i,i3}) = \frac{\sin\Theta_{i2,i3}}{\sin\Theta_{i,i3}}\sin\Phi_{i2,i3}. \tag{A.10}$$

The angles $\Theta_{i,i2}$, $\Phi_{i,i2}$ are given analogously to (A.4), (A.5) and (A.7) with $\Theta_{1,i3}\to\Theta_{1,i2}$, $\Phi_{1,i3}\to\Phi_{1,i2}$ and $\Theta_{i,i3}\to\Theta_{i,i2}$. We can also derive

$$\cos\Theta_{i,i2} = \cos\Theta_{i,j2}\cos\Theta_{j2,i2} + \sin\Theta_{i,j2}\sin\Theta_{j2,i2}\cos\Phi_{j2,i2}, \tag{A.11}$$
$$\sin\Phi_{i,i2} = -\sin(\Phi_{i,j2}-\Phi_{i,i2})\cos\Phi_{i,j2} + \cos(\Phi_{i,j2}-\Phi_{i,i2})\sin\Phi_{i,j2}, \tag{A.12}$$
$$\cos\Phi_{i,i2} = \sin(\Phi_{i,j2}-\Phi_{i,i2})\sin\Phi_{i,j2} + \cos(\Phi_{i,j2}-\Phi_{i,i2})\cos\Phi_{i,j2}, \tag{A.13}$$
$$\cos\Theta_{j2,j3} = \cos\Theta_{i,j2}\cos\Theta_{i,j3} + \sin\Theta_{i,j2}\sin\Theta_{i,j3}\cos(\Phi_{i,j2}-\Phi_{i,j3}), \tag{A.14}$$

where one has to insert

$$\sin(\Phi_{i,j2}-\Phi_{i,i2}) = \frac{\sin\Theta_{j2,i2}}{\sin\Theta_{i,i2}}\sin\Phi_{j2,i2}, \tag{A.15}$$
$$\cos(\Phi_{i,j2}-\Phi_{i,i2}) = \frac{\sin\Theta_{i,j2}\cos\Theta_{j2,i2} - \cos\Theta_{i,j2}\sin\Theta_{j2,i2}\cos\Phi_{j2,i2}}{\sin\Theta_{i,i2}}. \tag{A.16}$$

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